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MEMORANDUM FOR INDUSTRY DIRECTORS, LMSB
DIRECTOR, PREFILING AND TECHNICAL GUIDANCE, LMSB

FROM: Laura M. Prendergast /s/ Laura M. Prendergast
Director, Field Specialists

SUBJECT: Field Directive on the Use of Estimates from Probability Samples

The purpose of this memorandum is to establish guidelines for the Internal Revenue Service in evaluating samples and sampling estimates by taxpayers. This directive supersedes a prior document on the same subject issued in March 2002 and is intended to promote efficiency and consistency in the probability samples performed and examined by the IRS. The guidelines do not represent a technical position but provide audit issue direction to effectively utilize our resources. Further, as more fully described below, they do not replace or supersede specific statutory or regulatory requirements for substantiation or record keeping.

Examiners should perform a two-step inquiry in evaluating a taxpayer's probability sample. First, they should determine whether the taxpayer has appropriately used a probability sample to support or be the primary evidence of tax amounts. Second, they should determine whether the final answer represents a valid estimate.

The appropriateness of using a probability sample is a facts and circumstances determination. Some of the factors to be used in determining whether a probability sample is appropriate include the time required to analyze large volumes of data, the cost of analyzing data, and other books and records that may independently exist or have greater probative value.

Probability samples generally should be considered appropriate if there is a compelling reason for their use and taxpayers cannot reasonably obtain more accurate information. However, probability samples generally should not be considered appropriate if evidence is readily available from another source that can be demonstrated to be a more accurate answer, or if the use of sampling does not conform to Generally Accepted Accounting Principles (GAAP).

Once examiners determine that the use of a probability sample was appropriate, they should determine the validity of the final estimate. In general, an estimate from a taxpayer's sample should be considered valid (without regard to adjustment(s) based on audit issues) if all of the following conditions are met.

1. The taxpayer has maintained all of the proper documentation to support the statistical application, the sample unit findings and all aspects of the sample plan. This will generally include all of the information contained in Attachment A to these guidelines. The documentation requirement helps insure that the sample was conducted in a manner to support all the necessary elements of a probability sample.

2. The estimate is based on a probability (i.e., statistical) sample, where each sampling unit in the population has a known (non-zero) chance of selection, using either a simple random sampling method or stratified random sampling method.

3. The estimate is computed at the least advantageous 95% one-sided confidence limit. The "least advantageous" confidence limit is either the upper or lower limit that results in the least benefit to the taxpayer. However, if the relative precision for a sampling plan, as defined below, does not exceed 10%, the point estimate may be used in place of the least advantageous 95% one-sided confidence limit. Where the relative precision is less than 15% and greater than 10% the estimate will be computed as an amount between the least advantageous 95% one-sided confidence limit and the point estimate determined as follows:

   Estimate = Point Estimate (+ or -) (Relative Precision - .10)/.05 * (Point Estimate (+ or -) Least Advantageous 95% One-Sided Confidence Limit)

   Recognizing that many methods exist to estimate population values from the sample data, only the following estimators will be considered for acceptance. Variable estimators permitted include the Mean (also known as the direct projection method), Difference[1] (using "paired variables"), (combined) Ratio[1] (using a variable of interest and a "correlated" variable), and (combined) Regression[1] (using a variable of interest and a "correlated" variable). Since the latter two variable methods are statistically biased, it must be demonstrated that such bias is negligible before they will be considered

[1] The first variable used for the difference, ratio and regression estimators must be the variable used in the mean estimator. The second variable used for the difference, ratio and regression estimators must be a variable that can be paired with the first variable and should be related to the first variable. For example, in a typical audit sampling situation, the first variable would be the audited value of a transaction and the second variable would be the originally reported value of the same transaction.

acceptable. The formulas for these estimators have been provided in Technical Appendix to these guidelines and assume sampling without replacement. Attribute estimators permitted include (combined) proportion or total count.

(a) **Variable Sampling Plans**.

1) Of all the final estimates determined as qualifying, the estimate with the smallest overall standard error, as an absolute value, will generally be used (i.e., the size of the estimate is irrelevant in the determination of the value to be reported). Some situations exist where only a single estimator may be appropriate for the plan objective, such as when estimating a LIFO index, where only a Ratio estimation method may be appropriate. In those specialized situations, the relevant estimator may be evaluated without consideration of other methods.

2) Confidence limits are calculated by adding and subtracting the precision of the estimate from the point estimate where precision is determined by multiplying the standard error by (i) the 95% one-sided confidence coefficient based on the Student's *t*-distribution with the appropriate degrees of freedom, or (ii) 1.645 (i.e., the normal distribution), assuming the sample size is at least 100 in each non-100% stratum.

3) For either the (combined) Ratio or Regression methods, to demonstrate little statistical bias exists, the following applies after excluding all strata tested on a 100% basis (i.e., the entire population of a stratum is selected for evaluation).

   (i) The total sample size of all strata must be at least 100 units.

   (ii) Each stratum for which a population estimate is made should contain at least 30 sample units.

   (iii) The coefficient of variation of the paired variable[2] must be 15% or less.

   (iv) The coefficient of variation of the primary variable of interest, represented by either the corrected value[3] or the difference

2 [Standard Error of the Total "y" Variables] / [Point Estimate of the Total "y" Variables]. Where the "y" variables are commonly the reported values in accounting situations.

3 [Standard Error of the Total "x" Variables] / [Point Estimate of the Total "x" Variables]. Where the "x" variables are commonly the corrected values in accounting situations.

between the reported and corrected values[4] in common accounting situations, must be 15% or less.

(v) For only the (combined) Ratio method the reported values of the units must be of the same sign.

4) The relative precision for each estimator is commonly calculated by dividing the precision at the 95% one-sided confidence limit (sometimes referred to as sampling error) of the estimate by the estimate. Where an estimator may be calculated using either a corrected value or difference perspective, as in the case of Ratio and Regression methods or solely a corrected value perspective as in the case of a Mean method, the test will be applied on the basis of a difference perspective. In such cases the numerator of the calculation is the sampling error of the adjustment and the denominator the point estimate of the adjustment.

5) For specialized situations, such as when determining a LIFO index using probability sampling techniques, the 10% test that applies to the particular sampling objective, must be appropriate for the plan, and adjusted accordingly to reflect an acceptable level of precision. For a LIFO index the 10% test is determined by dividing the sampling error of the index by the point estimate of the index minus one, using the difference between the beginning and end of year sample unit values. Additional modifications may be necessary for other unique types of sampling plans.

6) For the purpose of the 10% relative precision test, any stratum where the sampling units or the process of evaluating the sampling units are different from those in other strata must be excluded in calculating the relative precision.

(b) Attribute Sampling Plans:

1) When using simple random samples, the confidence limits will be determined using the Hypergeometric, Poisson, or Binomial distribution. If the proportion being estimated is between 30% and 70%, then the normal distribution approximation may be used in lieu of one of the above distributions.

2) For stratified random samples, when at least two strata are sampled (i.e., not 100% samples), the confidence limits must be determined

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4 The smaller of [Standard Error of the Total "x-y" or Total "d" Variables] / [(Total Population Value Represented by "Y") + (Point Estimate of the Total "x-y" or Total "d" Variables)] or [Standard Error of the Total "x-y" or Total "d" Variables] / [(Point Estimate of the Total "x-y" or Total "d" Variables)]. Where the "x-y" variables are commonly represented by the difference ("d") between the corrected ("x") values and reported ("y") in accounting situations.

using the normal distribution approximation. Otherwise, item one above applies.

3) For the normal distribution approximation, the precision is calculated by multiplying the standard error by (i) the 95% one-sided confidence coefficient based on the Student's *t*-distribution with the appropriate degrees of freedom, or (ii) 1.645 (i.e., the normal distribution), assuming the sample size is at least 100 in each non-100% stratum.

4) One of the following two tests must be achieved for the use of the point estimate from an attribute sampling plan.

   - A relative precision of 10% or less must be achieved on the point estimate (i.e., the estimated proportion, p) and on its complement (i.e., 1 – p).

   - A simple random sample size of at least 300 must be used to determine the point estimate. The sample size of 300 excludes dummy and null sampling units.

The allowance of a taxpayer's estimate does not correspondingly require acceptance of the taxpayer's use of such estimate for the determination of associated adjustments, allocation, or subdivision of the findings for other purposes unless statistically determined according to these guidelines and applied on a basis appropriate for the circumstances. These guidelines address only the statistical requirements that must be met for a probability sample to meet preliminary acceptance and are not intended to further require acceptance of individual sample unit determinations. Valuation or attribute determinations remain subject to independent verification along with other non-statistical issues such as missing sampling items. Likewise, the statistical procedures followed may be examined and adjusted when discovered in error. Corrections to statistical methodology are permitted where possible to place the method in compliance with these guidelines. Any fatal error in statistical methodology which renders the probability sample invalid will preclude the use of any statistical estimate based on the sample and will only allow for consideration of the sample findings on an actual basis. Where a probability sample is determined to be not appropriate and raised as an issue, the examining agent may pursue a more accurate determination or allow the findings of units examined on an actual basis. However, the computational validity of the estimator should still be considered and addressed along with other alternative issues in unagreed cases.

This memorandum is not intended to supersede any formal regulations, rulings, or procedures (e.g., Rev. Proc. 2007-35, 2007-1 C.B. 1349, and Rev. Proc. 2004-29, 2004-1 C.B. 918) that address the specific application of statistical principles. It is recognized that existing industry practices and specific taxpayers may be using techniques that are not covered by this directive or other published documents. If a taxpayer has employed a probability sample or method not covered, the estimate will be referred to a Statistical Sampling Coordinator for resolution or issue development. Similarly, the application of probability sampling techniques to unique areas, like for LIFO inventory as covered earlier, may require modification of the guidelines to better fit the circumstances, and as a result should also be referred for consideration.

These guidelines do not relieve taxpayers of their responsibility to maintain any documentation required by section 6001 of the Internal Revenue Code, other sections, or subsections, which have specific documentation requirements, for the entire population. Issues regarding documentation or support may be raised as appropriate.

This LMSB Directive is not an official pronouncement of the law or the Service's position and cannot be used, cited, or relied upon as such.

Attachment (1)

cc: Commissioner, LMSB
Deputy Commissioner, Operations
Deputy Commissioner, International
Director, Planning, Quality, Analysis & Support
Director, Research & Workload Identification
Division Counsel, LSMB
Chief, Appeals
Commissioner, SBSE

ATTACHMENT A

Probability Sample Documentation Standards

Sampling Plan

A written sampling plan should be prepared and formalized prior to the execution of the sample. A plan would include the following:

- The objective of the plan including a description of what value is being estimated and for which tax year(s) the estimate is applicable.
- Population definition and reconciliation of the population to the tax return.
- Definition of the sampling frame.
- Definition of the sampling unit.
- Source of the random numbers, the starting point or seed, and the method used in selecting them.
- Sample size, along with supporting factors in the determination.
- Method used to associate random numbers to the frame.
- Steps to be taken to insure that the serialization of the frame is carried out independent of the drawing of random numbers.
- Steps to be taken in evaluating the sampling unit.
- The appraisal method(s) to be used in appraising the sample.

Sample Execution Documentation

The execution of the sample must be documented and include information for each of the following:

- The seed or starting point of the random numbers.
- The pairing of random numbers to the frame along with supporting information to retrace the process.
- List of the sampling units selected and the results of the evaluation of each unit.
- Supporting documentation such as notes, invoices, purchase orders, project descriptions etc., which support the conclusion reached about each sample item.
- The calculation of the projected estimate(s) to the population, including the computation of the standard error of the estimate(s).
- A statement as to any slips or blemishes in the execution of the sampling procedure and any pertinent decision rules.
- Computation of all associated adjustments. (An example of an associated adjustment would be the amount of depreciation allowable based on a probability determination of an amount capitalized).

TECHNICAL APPENDIX

FORMULAS

UNSTRATIFIED (SIMPLE RANDOM SAMPLE) MEAN ESTIMATOR	STRATIFIED MEAN ESTIMATOR

Sample Mean of Audited Amounts

$$\bar{x} = \frac{\sum x_j}{n}$$

Estimate of the Total Audited Amount

$$\hat{X}_M = N\bar{x}$$

$$\hat{X}_{M_s} = \sum (N_i \bar{x}_i)$$

Estimated Standard Deviation of Audited Amounts

$$S_X = \sqrt{\frac{[\sum (x_j^2)] - n(\bar{x}^2)}{n-1}}$$

Estimated Standard Error of the Total Audited Amount

$$\hat{\sigma}(\hat{X}_M) = \frac{NS_x\sqrt{1 - n/N}}{\sqrt{n}}$$

$$\hat{\sigma}(\hat{X}_{M_S}) = \sqrt{\sum \left[N_i (N_i - n_i) \frac{S_{x_i}^2}{n_i} \right]}$$

Achieved Precision of the Total Audited Amount

$$A'_M = \frac{NU_R S_x \sqrt{1 - n/N}}{\sqrt{n}}$$

$$A'_{M_S} = U_R \sqrt{\sum \left[N_i (N_i - n_i) \frac{S_{x_i}^2}{n_i} \right]}$$

TECHNICAL APPENDIX

FORMULAS

UNSTRATIFIED (SIMPLE RANDOM SAMPLE) DIFFERENCE ESTIMATOR	STRATIFIED DIFFERENCE ESTIMATOR

Estimate of the Total Difference

$$\hat{D} = N\bar{d}$$

$$\hat{D}_S = \sum (N_i \bar{d}_i)$$

Estimate of the Total Audited Amount

$$\hat{X}_D = Y + \hat{D}$$

$$\hat{X}_{D_S} = Y + \hat{D}_S$$

Estimated Standard Deviation of Difference Amounts

$$S_D = \sqrt{\frac{[\sum (d_j^2)] - n(\bar{d}^2)}{n-1}}$$

Estimated Standard Error of the Total Difference Amount

$$\hat{\sigma}(\hat{D}) = \frac{NS_D\sqrt{1 - n/N}}{\sqrt{n}}$$

$$\hat{\sigma}(\hat{D}_S) = \sqrt{\sum \left[N_i (N_i - n_i) \frac{S_{D_i}^2}{n_i} \right]}$$

Achieved Precision of the Total Difference Amount

$$A'_D = \frac{NU_R S_D \sqrt{1 - n/N}}{\sqrt{n}}$$

$$A'_{D_S} = U_R \sqrt{\sum \left[N_i (N_i - n_i) \frac{S_{D_i}^2}{n_i} \right]}$$

TECHNICAL APPENDIX

FORMULAS

UNSTRATIFIED (SIMPLE RANDOM SAMPLE) RATIO ESTIMATOR	**STRATIFIED COMBINED RATIO ESTIMATOR**

Estimated Ratio of Audited Amount to Recorded Amount

$$R = \frac{\sum x_j}{\sum y_j} = 1 + \frac{\sum d_j}{\sum y_j}$$

$$\hat{R}_c = \frac{\sum (N_i \bar{x}_i)}{\sum (N_i \bar{y}_i)} = 1 + \frac{\sum (N_i \bar{d}_i)}{\sum (N_i \bar{y}_i)}$$

Estimate of the Total Audited Amount

$$\hat{X}_R = Y\hat{R}$$

$$\hat{X}_{R_C} = Y\hat{R}_C$$

Estimated Standard Deviation of the Ratio

$$S_R = \sqrt{\frac{\sum (x_j^2) + \hat{R}^2 \sum (y_j^2) - 2\hat{R} \sum (x_j y_j)}{n-1}}$$

Estimated Standard Deviation of the Ratio in *i*th Stratum

$$S_{Rc_i} = \sqrt{\frac{[(\sum x_{i_j}^2 - (\sum x_{i_j})^2 / n_i)] + [\hat{R}_c^2 (\sum y_{i_j}^2 - (\sum y_{i_j})^2 / n_i)] - [2\hat{R}_c (\sum x_{i_j} y_{i_j} - n_i \bar{x}_i \bar{y}_i)]}{n_i - 1}}$$

Estimated Standard Error of the Total Audited Amount

$$\hat{\sigma}(\hat{X}_R) = \frac{N S_R \sqrt{1 - n/N}}{\sqrt{n}}$$

$$\hat{\sigma}(\hat{X}_{R_C}) = \sqrt{\sum \left[N_i (N_i - n_i) \frac{S_{Rc_i}^2}{n_i} \right]}$$

Achieved Precision of the Total Audited Amount

$$A'_R = \frac{N U_R S_R \sqrt{1 - n/N}}{\sqrt{n}}$$

$$A'_{Rc} = U_R \sqrt{\sum \left[N_i (N_i - n_i) \frac{S_{Rc_i}^2}{n_i} \right]}$$

TECHNICAL APPENDIX

FORMULAS

UNSTRATIFIED (SIMPLE RANDOM SAMPLE) REGRESSION ESTIMATOR	**STRATIFIED COMBINED REGRESSION ESTIMATOR**

Estimated Regression Coefficient

$$b = \frac{[\sum(x_j y_j)] - n\bar{x}\bar{y}}{[\sum(y_j^2)] - n(\bar{y}^2)} = 1 + \frac{[\sum(d_j y_j)] - n\bar{d}\bar{y}}{[\sum(y_j^2)] - n(\bar{y}^2)}$$

$$b_c = \frac{\sum N_i(N_i - n_i) S_{XY_i} / n_i}{\sum N_i(N_i - n_i) S_{Y_i}^2 / n_i} = 1 + \frac{\sum N_i(N_i - n_i) S_{DY_i} / n_i}{\sum N_i(N_i - n_i) S_{Y_i}^2 / n_i}$$

Estimate of the Total Audited Amount

$$\hat{X}_G = N\bar{x} + b(Y - N\bar{y})$$

$$\hat{X}_{Gc} = \sum(N_i \bar{x}_i) + b_c\left[Y - \sum(N_i \bar{y}_i)\right]$$

Estimated Standard Deviation of the Regression Amounts

$$S_G^* = \sqrt{\frac{1}{n-2}\left[[\sum(x_j^2)] - n(\bar{x}^2) - \frac{(\sum(x_j y_j) - n\bar{x}\bar{y})^2}{\sum(y_j^2) - n(\bar{y}^2)}\right]}$$

*n-2 for unstratified, n-1 for stratified

Estimated Covariance between the Audited and Recorded Amounts in *i*th Stratum

$$S_{XY_i} = \frac{[\sum(x_{i_j} y_{i_j})] - n_i \bar{x}_i \bar{y}_i}{n_i - 1}$$

Estimated Standard Deviation between the Audited and Recorded Amounts in *i*th Stratum

$$S_{Gc_i} = \sqrt{s_{x_i}^2 - 2b_c S_{XY_i} + b_c^2 s_{Y_i}^2}$$

Estimated Standard Error of the Total Audited Amount

$$\hat{\sigma}(\hat{X}_G) = \frac{N S_G \sqrt{1 - n/N}}{\sqrt{n}}$$

$$A'_{Gc} = U_R \sqrt{\sum\left[N_i(N_i - n_i)\frac{S_{Gc_i}^2}{n_i}\right]}$$

Achieved Precision of the Total Audited Amount

$$A'_G = \frac{N U_R S_G \sqrt{1 - n/N}}{\sqrt{n}}$$

$$\hat{\sigma}(\hat{X}_{G_C}) = \sqrt{\sum\left[N_i(N_i - n_i)\frac{S_{Gc_i}^2}{n_i}\right]}$$

TECHNICAL APPENDIX

Definition of Symbols

TERM	DEFINITION
n	**Sample Size**
N	**Population Size**
x	**The value of the sampling unit that is being used as the primary variable of interest. In audit sampling, this would be the audited (or revised) value of the transaction. In LIFO Index samples, it is represented by the end of year value.**
y	**The value of the sampling unit that is being used as the "paired" variable that is related to the variable of interest. In audit sampling, this would be the reported (or original) value of the transaction. For LIFO Index samples, it is represented by the beginning of year value. This variable is used with the difference, ratio, and regression estimators.**
d	**The value of the sampling unit that is the difference between "paired" variable (y) and the variable of interest (x). That is,** $d = x - y$ **In audit sampling, this would be the difference (or the change) of each transaction's value.**
X	**The total value of the primary variable of interest. In audit sampling, this would be the estimated total audited value of the population. Typically, this value is not known for the entire population and is estimated based on the probability sample selected.**
Y	**The total value of the variable that is paired with variable of interest. In audit sampling, this would be the total reported value of the population. Typically, this value is known for the entire population and may be estimated based on the probability sample selected.**
D	**The total value of the difference between the "paired" variable and the variable of interest. In audit sampling, this would be the estimated total difference of the population. Typically, this value is not known for the entire population and is estimated based on the probability sample selected.**
U_R	**The confidence coefficient which is based on either the Student's *t*-distribution or the normal distribution. For example, a 95% one-sided confidence coefficient based on the normal distribution is 1.645. This term is often referred to as the *t*-value and the *z*-value.**