

Estimating the Wealth of Top Wealth-Holders From Estate Tax Returns

by James D. Smith, Ph.D

Introduction

The estate multiplier technique, as currently used, rests on the assumption that death draws a random sample, stratified by age and sex, of the living population. If one has available age-sex-specific mortality rates, an estimate of the wealth of the living in a given period of time can be made by stepping-up the wealth of decedents in each age-sex class by the inverse of the mortality rate associated with that age-sex class, and summing the results across all age-sex classes:

$$W_t = \sum_{i=1}^m \sum_{j=1}^2 \frac{V_{ij}}{M_{ij}} w_{ij}$$

Where V_{ij} is the number of living persons, M_{ij} is the number of decedents, and w_{ij} is the wealth of decedents, all associated with the ij th age-sex class. The term V_{ij}/M_{ij} is the estate multiplier for the ij th age-sex class. The technique can also be used to derive distributions for the living by any identifiable characteristic of the decedents or their wealth.

Using the Federal estate tax returns, the Internal Revenue Service is preparing estate multiplier estimates of the wealthiest strata of the United States population in 1962. This is gratifying to economists looking toward comprehensive estimates of U.S. wealth and national balance sheets by the end of the decade of the '60's.

This paper presents (1) a short discussion of the problems and limitations of estate multiplier estimates, (2) a brief review of previous estate multiplier estimates, (3) some results from a 1958 estimate, and (5) some ideas for future estate multiplier estimates (by no means a plan for their realization, in view of the demand for other statistics). A technical appendix demonstrates

mathematically the derivation of estimates by use of the estate multiplier technique.

Problems and Limitations

Administrative records used for the collection of death taxes have been employed in England, Wales, Scotland, Australia, New Zealand and the United States as inputs to the estate multiplier.

As is almost always the case, however, administrative records reflect the immediate needs of administrators; only indirectly the needs of scholars -- and often policy makers. Death tax returns, for instance, are required only for decedents with estates above a rather high level of wealth. In the United States the present filing requirement is very high, \$60,000. (We hereafter refer to persons with gross assets of over \$60,000 as top wealth-holders.)

Because of the high filing requirement, the estate multiplier method is not suited to estimating total private wealth in the United States or its distribution along a complete Lorenz curve. At the upper levels, however, it enjoys a comparative advantage over field surveys. In 1963, for instance, 78,393 estate tax returns were filed for persons with assets of over \$60,000. The Federal Reserve Board, on the other hand, used a total sample of only 3,600 families to estimate the distribution of wealth for the entire population.¹

But even with the very large sample of top wealth-holders provided by estate tax returns, considerable sampling variability attaches to estimates of ownership of assets such as municipal and corporate bonds, the distributions of which are highly skewed even among the rich. Further, the sampling variability attaching to estimates of wealth held by persons of younger ages is greater than corresponding estimates for older persons because death draws a much thinner sample of the

young.

But beyond the general problems associated with any sample, three areas of difficulty are inherent in the application of the estate multiplier to U.S. estate tax returns. First, there is the problem of selecting the appropriate set of mortality rates; second, that of informational gaps in the returns; and third, the necessity to infer year of death.

If mortality rates and wealth are correlated, then a biased estimate will be produced using mortality rates unadjusted for wealth level. Indirect evidence supports the view that an inverse correlation exists -- the rich do live longer.² Under this circumstance the estate multipliers will be too low, and under-state the wealth of top wealth-holders.

The major information gaps in the U.S. estate tax returns are missing ages and the absence of life insurance cash surrender value and of post audit asset valuations.

Each year a number of returns are filed without age information (1,383 out of 55,685 in 1958). It is necessary, therefore, either to exclude the returns for decedents of unspecified age or to impute ages.

Life insurance in the estates of the decedents, unlike other assets, is not isomorphic with that in the hands of the living. The economic value of a life insurance contract to a living person is its cash surrender value, but death brings the face value of the contract into the estate. In order to estimate the wealth of top wealth-holders, it is necessary to adjust downward the life insurance component reported on estate tax returns.

Although evidence suggests that auditing may increase asset values reported on estate tax returns by as much as 10 percent, only tabulations of unaudited returns have been available for estate multiplier estimates.³ Under-estimation here, however, is probably less than in field surveys.

For administrative purposes it makes sense to tabulate returns by calendar year of filing. Conse-

quently, one does not have the sample death drawn of the living population in a calendar year; rather returns filed in a calendar year, which include returns for deaths of several years. The executor has 15 months after decedent's date of death in which to file a return, and extensions may be granted beyond that. During 1965 returns filed within the 15 month filing period may be for decedents whose date of death was as early as October 1963. On the other hand, returns for some 1964 decedents will not be filed until 1966. A few returns filed in 1965 with extensions of time may be for decedents whose date of death was over 10 years ago.

Although the first study of the temporal distribution of dates of death will be done with the 1966 filings, it is believed, on the basis of limited evidence, that the great majority of returns filed in a given calendar year represent deaths in the preceding year. In all estate multiplier estimates it has been inferred that a return filed in a given year represented death in the preceding year. Because the number of returns and asset prices have moved secularly upward, the inference of the year of death probably introduces a downward bias in the wealth estimate.

Previous Uses of the Estate Multiplier Technique

Although financial information about decedents was used to estimate total wealth of the living by the "interval devolution" method in the 19th century, the estate multiplier technique was first suggested by Coughlin in 1906.⁴

The first estate multiplier estimate was made by Bernard Mallet⁵ in 1908 when he applied inverse mortality rates to English estate duty tabulations for 1905 and 1906 and the second was apparently made by Laughton for Victoria, Australia for 1911-12.⁶

In 1918 G. H. Knibbs published estimates for Australia for the years between 1878 and 1911.⁷ He used age-sex specific mortality rates, but his data did not provide a break-down by asset type. He was well aware of the possibility that mortality rates might be inversely correlated with wealth, but after an examination of life insurance records,

decided the evidence for Australia did not support reducing the age-sex-specific mortality rates.

Estate multiplier estimates for England and Wales for the years 1911 to 1913, 1924 to 1930 and 1936, were produced by Daniels and Campion.⁸ They contended the £100 filing exclusion filtered lower social classes out of the sample. Comparing occupations of decedents for whom estate duty returns had been filed with the occupational distribution in the Decennial Supplement on Occupational Mortality of the 1921 British Census of Population, they found the majority of decedents with more than £100 were in the top social classes used in the census. They then pointed to the significantly lower mortality rates for each of the upper classes in each of the four age brackets spanning the age interval 25 to 65. On the basis of this evidence, they lowered the mortality rates (thus raising the multipliers used for their estimate).

Kathleen Langly has published estimates for England, Scotland and Wales for 1936-38, 1946-47, and 1950-51.⁹ In each case she used general age-sex-specific mortality rates unadjusted for social class. An estimate for England for 1947-49 by A. M. Cartter, however, did employ age-sex-specific mortality rates so adjusted.¹⁰

Lydall and Tipping, using age-sex-specific mortality rates for the top two social classes defined in Great Britain's 1951 Census of Population, have made estimates for each of the years 1951 through 1958 for England, Scotland and Wales (England and Wales together for each year except 1957 and 1958).¹¹ As with prior estate multiplier estimates for Great Britain, they were not able to directly estimate the asset composition of wealth because Her Majesty's Commissioners did not publish tabulations of asset type by age and sex of decedent. Lydall and Tipping imputed an asset composition on the basis of the size distribution of estates within each age-sex cell.

To dampen the effect of sampling errors, they averaged the estimates of wealth above £2000 for the years 1951 through 1956, taking the result as their best estimate for beginning of year 1954 for persons with wealth of £2000 and over.

The first British estate multiplier estimate to get directly at the composition of wealth was done by J.R.S. Revell for 1957-58. His findings show that Lydall and Tipping's imputation of asset composition based entirely on size distributions was far from correct.¹² An official estimate for Great Britain for 1960, also produced a direct estimate of the composition of the wealth of the wealthy.¹³

Two estate multiplier studies previous to the 1958 results presented below have been made for the United States. In the first, Mendershausen made an estimate for each of the years 1922, 1924, 1941, 1944 and 1946.¹⁴ For the year 1944 a special tabulation of gross estate by type of asset and age of decedent was prepared for him by Internal Revenue Service. Unfortunately, the tabulations did not provide sex of decedents. Using both white age-specific mortality rates and a set of rates adjusted for social class, Mendershausen estimated the wealth of top wealth-holders by type of asset. The set of social class adjusted rates were based on the experience of the Metropolitan Life Insurance Company with a group of risks called the "\$5,000 whole life classification." composed predominantly of well-to-do individuals.¹⁵

With a tabulation by asset type, Mendershausen was able to isolate life insurance and adjust for the difference between the value of proceeds reported in the estate of a decedent and cash surrender value the instant before death. To do this, he obtained from "one fairly large" insurance company the ratios of reserves to face values by age groups of policyholders. It was basically this set of ratios which he used to reduce the proceeds reported on estate tax returns to estimate the life insurance equity of top wealth-holders in 1944. Mendershausen excluded those returns on which age of decedent was not reported.

Lampman estimated the wealth of top wealthholders in 1953.¹⁶ As did Mendershausen, he worked with a special Internal Revenue Service tabulation. Decedents were classified by State and marital status, as well as age and sex. Assets were classified by 10 types (including debt). He used

social class mortality rates based primarily on a study of differential mortality by Moriyama and Guralnick and the mortality experience of a large insurance company with a group of risks buying relatively large policies.¹⁷

Life insurance proceeds reported in the estates of decedents were reduced by a set of ratios of reserves to face values by age of policyholder. The ratios were obtained by reducing Mendershausen's ratios on the ground that the overall ratio of reserves to face value had decreased since 1944.

Lampman imputed to decedents of unknown age the average age of all decedents, and for the first time related U.S. estimates to national balance sheets.

The 1958 Estimates

The estimates for 1958 which follow are from a study by Smith of the income and wealth of top wealth-holders in 1958.¹⁸ The mortality rates used were obtained by reducing the 1958 white age-sex-specific mortality rates, based on post 1960 Census population estimates, by the same proportion Lampman had reduced the comparable rates for 1953. Insurance was reduced by the same ratios used by Lampman. Decedents of unknown age were imputed the average age of all decedents of the same sex in the same size of gross estate class. Table 1 shows the composition by type of property and size of gross estate of the wealth of top wealth-holders in 1958.

Because one of the important uses of estate multiplier estimates is to gauge the concentration of wealth, a mid-year 1958 national balance sheet for individuals has been constructed. The balance sheet was constructed by modifying Goldsmith's balance sheet for nonfarm households to include noncorporate farms and to exclude nonprofit institutions. Goldsmith's balance sheet values are for the end-of-year.¹⁹ His estimates were converted to mid-1958 by taking the arithmetic mean of his end-of-year 1957 and 1958 values. This was done to achieve correspondence with the wealth estimates which are based on a sample of decedents assumed to have been drawn rather evenly

over the year 1958.

The method of constructing the balance sheet for individuals differed slightly from that used by Lampman.²⁰ Because of this, and also because Lampman worked with preliminary data, a new balance sheet for individuals was constructed for mid-1953 to permit a direct comparison of the share of top wealth-holders in personal wealth based on his finding for 1953 and Smith's for 1958. The revision of the 1953 balance sheet had slight effect on the share of top wealth-holders as reported by Lampman.

Table 2 shows the 1953 and 1958 balance sheets. In accordance with the concepts adopted by Lampman, an individual sector was constructed to show both prime wealth and total wealth. Total wealth includes all personal wealth from which one receives "direct" benefits. Thus, it includes pension and trust funds though their corpus may not be subject to invasion. Prime wealth is total wealth less the value of assets in trust funds and pension reserves.

Because of one extremely unusual case which appeared in the Internal Revenue printouts, some adjustment to the composition of assets of top wealth-holders is called for. One female under 40 years of age was tabulated as having an estate of \$14,526,000. Examination of the detailed IRS tabulations shows that \$13,609,000 of this estate was in annuities. Blowing-up this value by the high multiplier associated with females under 40, yields an estimate of annuities of \$19.8 billion, a very dubious value.²¹

Assuming the gross estate of this one decedent was distributed among assets in the same proportions as assets in the aggregate gross estate of all other top wealth-holders in the same estate size class, then the distribution of assets and their respective shares appears as shown in table 3. Adjusting the total composition shown in table 1 for this one case results in the estimated concentration of State and local bonds, other bonds and corporate stock being considerably increased; and that of "miscellaneous assets" decreased.

In order to place the share of wealth owned

Table 1. COMPOSITION OF GROSS ESTATE BY SIZE OF GROSS ESTATE FOR TOP WEALTH-HOLDERS IN 1958

Size of Gross Estate *	Number of top wealth-holders (Thousands)	Type of Property							
		Real estate	Federal bonds	State and local bonds	Other bonds	Corporate stock	Cash	Notes and mortgages	
		(2)	(3)	(4)	(5)	(6)	(7)	(8)	
Total	2,595	114.4	16.2	10.6	3.5	182.0	39.3	17.6	
\$60,000 under \$70,000	274	6.2	0.8	0.1	0.1	3.6	2.5	0.6	
\$70,000 under \$80,000	303	7.7	0.9	0.1	0.1	4.6	2.7	0.8	
\$80,000 under \$90,000	241	7.0	0.8	a	0.1	4.0	2.2	0.7	
\$90,000 under \$100,000	211	6.5	0.8	a	0.1	4.4	2.0	0.6	
\$100,000 under \$120,000	357	12.8	1.4	0.1	0.2	8.7	4.1	1.4	
\$120,000 under \$150,000	350	13.6	1.6	0.1	0.2	11.8	4.5	1.7	
\$150,000 under \$200,000	310	14.2	1.9	0.2	0.3	14.7	4.6	2.4	
\$200,000 under \$300,000	269	15.4	2.2	0.6	0.5	21.8	5.2	2.6	
\$300,000 under \$500,000	151	11.8	1.8	1.0	0.6	24.0	3.9	2.1	
\$500,000 under \$1,000,000	88	9.8	1.8	2.1	0.6	29.9	3.7	2.1	
\$1,000,000 under \$2,000,000	28	4.2	1.2	2.3	0.3	21.2	1.7	1.5	
\$2,000,000 under \$3,000,000	6	1.4	0.3	1.1	0.1	9.1	0.5	0.4	
\$3,000,000 under \$5,000,000	2	1.2	0.2	1.5	0.2	5.3	0.5	0.2	
\$5,000,000 under \$10,000,000	2	0.7	0.3	0.9	0.1	6.1	0.7	0.2	
\$10,000,000 under \$20,000,000	2	1.5	a	0.4	a	9.9	0.3	0.2	
\$20,000,000 or more	b	0.2	0.1	0.4	a	2.9	0.1	0.2	

Detail may not add to total because of rounding.
Notes at end of table

Table 1. COMPOSITION OF GROSS ESTATE BY SIZE OF GROSS ESTATE FOR TOP WEALTH-HOLDERS IN 1958 -- Continued

Size of Gross Estate *	Type of Property -- Continued				Gross Estate		Debts and mortgages	Economic estate **
	Life insurance		Annuities	Miscellaneous assets	Using face value of life insurance	Using equity value of life insurance		
	Face value	Equity value						
(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	
Total	52.4	13.3	19.8	55.2	511.1	472.0	42.9	429.0
\$60,000 under \$70,000	1.9	0.5	0.1	2.0	17.8	16.4	1.2	15.2
\$70,000 under \$80,000	3.1	0.7	0.1	2.6	22.7	20.3	1.7	18.6
\$80,000 under \$90,000	3.2	0.7	0.1	2.3	20.5	18.0	1.5	16.4
\$90,000 under \$100,000	3.2	0.8	0.1	2.2	20.1	17.6	1.6	16.0
\$100,000 under \$120,000	5.9	1.4	0.2	4.3	39.0	34.5	3.3	31.2
\$120,000 under \$150,000	7.6	1.8	0.1	5.4	46.9	41.0	4.5	36.5
\$150,000 under \$200,000	8.6	2.0	0.2	6.3	53.4	46.8	5.0	41.8
\$200,000 under \$300,000	8.0	2.1	0.3	8.1	64.6	58.7	6.1	52.7
\$300,000 under \$500,000	5.3	1.5	0.1	6.8	57.4	53.6	5.0	48.6
\$500,000 under \$1,000,000	3.7	1.2	0.2	5.7	59.6	57.2	5.2	51.9
\$1,000,000 under \$2,000,000	1.4	0.5	a	4.7	38.4	37.5	3.4	34.0
\$2,000,000 under \$3,000,000	0.3	0.1	a	2.0	15.3	15.1	1.2	13.9
\$3,000,000 under \$5,000,000	0.1	a	a	0.6	9.7	9.7	0.6	9.1
\$5,000,000 under \$10,000,000	0.1	a	a	1.7	10.6	10.6	0.8	9.7
\$10,000,000 under \$20,000,000	a	a	++	0.6	31.3	31.3	1.8	29.5
\$20,000,000 or more	a	a	a	a	3.7	3.7	a	3.6

Detail may not add to total because of rounding.

a. Rounds to less than \$100,000,000

b. Rounds to less than 1,000

* Gross estate size distribution obtained by using the face value of life insurance.

** Gross estate (using equity value of life insurance) less Debts and mortgages, column 14 less column 15.

++ See footnote 21.

Table 2. DERIVATION OF NATIONAL BALANCE SHEET FOR INDIVIDUALS BY TYPE OF PROPERTY FOR MID-YEAR 1958 AND 1953

PART I -- MID-YEAR 1958

Type of Property	Nonfarm house hold	Farm house hold	Nonfarm noncor- porate businesses	Trust funds	Nonprofit institu- tions	Individuals	
						Total wealth	Prime wealth
(Billion dollars)							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Real estate, total	452.8	113.5	63.4	—	30.4	599.3	599.3
Residential structures	338.6	18.1	16.1	—	—	—	—
Nonresidential structures	25.3	15.8	24.8	—	25.3	—	—
Land	88.9	79.6	22.5	—	5.1	—	—
Federal bonds, total	59.7	4.9	—	2.7	1.7	62.9	60.2
Short-term	3.9	—	—	—	—	—	—
Savings bonds	43.5	4.9	—	—	—	—	—
Other Federal bonds	12.3	—	—	—	—	—	—
State and local bonds	24.1	—	—	7.8	0.6	23.5	15.7
Other bonds	11.0	—	—	2.9	3.2	7.8	4.9
Corporate stock, total	299.6	—	—	33.3	9.0	290.6	257.3
Preferred	10.4	—	—	1.5	0.8	—	—
Common	289.2	—	—	31.8	8.2	—	—
Cash, total	193.6	8.6	12.9	0.4	5.3	209.8	209.4
Currency and demand deposits	59.9	5.8	12.9	0.4	5.3	—	—
Other deposits	133.7	2.8	—	—	—	—	—
Notes and mortgages, total	28.9	—	16.1	0.7	0.4	44.6	43.9
Nonfarm mortgages:							
Residential	12.4	—	—	—	—	—	—
Nonresidential	9.4	—	—	—	—	—	—
Farm mortgages	4.4	—	—	—	—	—	—
Consumer credit	—	—	4.8	—	—	—	—
Trade credit	—	—	11.3	—	—	—	—
Other loans	2.7	—	—	—	—	—	—
Life insurance reserves	96.8	6.2	—	—	—	103.0	103.0
Pension and retirement funds:							
Private	25.0	—	—	—	—	25.0	—
Government	65.0	0.4	—	—	—	65.5	—
Miscellaneous assets, total	172.5	56.4	43.5	1.9	1.9	270.5	268.6
Equity in mutual financial institutions	7.7	—	—	—	—	—	—
Producer durables	2.0	17.4	26.7	—	1.9	—	—
Consumer durables	162.2	13.3	—	—	—	—	—
Inventories	—	22.2	16.8	—	—	—	—
Other intangible assets	0.6	3.5	—	0.6	—	—	—
Other tangible assets	—	—	—	1.3	—	—	—
Gross assets	1,429.1	190.0	135.9	49.7	52.5	1,702.5	1,562.3
Debt	170.2	19.0	39.1	—	5.5	222.8	222.8
Economic estate	1,258.9	171.0	96.8	49.7	47.0	1,479.7	1,339.5

Table 2. DERIVATION OF NATIONAL BALANCE SHEET FOR INDIVIDUALS BY TYPE OF PROPERTY FOR MID-YEAR 1958 AND 1953--Continued

PART II -- MID-YEAR 1953

Type of Property	Nonfarm house hold	Farm house hold	Nonfarm noncor- porate businesses	Trust funds	Nonprofit institu- tions	Individuals	
						Total wealth	Prime wealth
(Billion dollars)							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Real estate, total	330.1	91.5	49.0	—	20.3	450.3	450.3
Residential structures	254.8	16.0	14.8	—	—	—	—
Nonresidential structures	17.0	13.2	16.9	—	17.0	—	—
Land	58.3	62.3	17.3	—	3.3	—	—
Federal bonds, total	60.3	4.5	—	5.5	1.7	63.1	57.6
Short-term	1.8	—	—	—	—	—	—
Savings bonds	46.4	4.5	—	—	—	—	—
Other Federal bonds	12.1	—	—	—	—	—	—
State and local bonds	16.4	—	—	5.3	0.4	16.0	10.7
Other bonds	6.3	—	—	1.9	1.8	4.5	2.6
Corporate stock, total	162.9	—	—	19.6	5.1	157.8	138.2
Preferred	9.4	—	—	1.6	0.7	—	—
Common	153.5	—	—	18.0	4.4	—	—
Cash, total	141.8	8.3	10.4	0.6	3.9	156.6	156.0
Currency and demand deposits	56.7	6.0	10.4	0.6	—	—	—
Other deposits	85.1	2.3	—	—	—	—	—
Notes and mortgages, total	20.4	—	12.8	0.8	0.3	32.9	32.1
Nonfarm mortgages:							
Residential	9.4	—	—	—	—	—	—
Nonresidential	6.2	—	—	—	—	—	—
Farm mortgages	3.1	—	—	—	—	—	—
Consumer credit	—	—	4.1	—	—	—	—
Trade credit	—	—	8.7	—	—	—	—
Other loans	1.7	—	—	—	—	—	—
Life insurance reserves	70.4	4.9	—	—	—	75.3	75.3
Pension and retirement funds:							
Private	10.5	—	—	—	—	10.5	—
Government	50.3	0.5	—	—	—	50.8	—
Miscellaneous assets, total	123.7	51.6	33.2	1.9	1.0	207.5	205.6
Equity in mutual financial institutions	4.8	—	—	—	—	—	—
Producer durables	1.0	15.7	18.2	—	1.0	—	—
Consumer durables	117.3	13.2	—	—	—	—	—
Inventories	—	20.0	15.0	—	—	—	—
Other intangible assets	0.6	2.7	—	0.6	—	—	—
Other tangible assets	—	—	—	1.3	—	—	—
Gross assets	993.1	161.3	105.4	35.6	34.5	1,225.3	1,128.4
Debt	98.3	14.1	24.6	—	3.2	133.8	133.8
Economic estate	894.8	147.2	80.8	35.6	31.3	1,091.5	994.6

Notes for Table 2

Column 1, Nonfarm households, is the average of end-of-year asset values: for 1953, end-of-year, 1952 and 1953; for 1958, end-of-year 1957 and 1958 from Goldsmith, Studies in the National Balance Sheet, Vol. II, pp. 118f. Nonfarm noncorporate business assets of individuals are not included in column 1, but shown as a separate sector, Nonfarm Noncorporate Businesses, in column 3.

Column 2, Farm households, was derived by averaging Goldsmith's year-end values for his agriculture sector, Studies in the National Balance Sheet, Vol. II, pp. 132f: for 1953, end-of-year 1952 and 1953; for 1958, end-of-year 1957 and 1958. The values obtained were then reduced by 5 percent to eliminate corporate farms. The basis for this adjustment is Mary M. B. Harmon, A Statistical Summary of Farm Tenure, Agriculture Research Service, U.S.D.A., 1958, p. 2, which shows five percent of farm acreage was owned by corporations in 1954. Discussions with personnel of the Department of Agriculture and the Bureau of the Census who deal with agricultural data cast doubt upon the assumption that only five percent of the value of farm assets is owned by corporations. It is suspected that the five percent of farm acreage owned by corporations is above average in value and that corporate farms are more capital intensive than the average. However, for lack of hard data to support a further reduction in Goldsmith's agriculture sector, assets were reduced by only five percent.

Column 3, Nonfarm noncorporate businesses, is an average of Goldsmith's end-of-year asset values for such businesses from Studies in the National Balance Sheet, Vol. II, pp. 126f.

Column 4: The assets and liabilities of trust funds, all of which are included in the nonfarm household sector, are listed separately here as the trust sector. Trust funds, for 1958 is an average of common trust funds for 1957 and 1958 from Goldsmith, Studies in the National Balance Sheet, Vol. II, pp. 122f, plus the values for personal trust funds from the "Report of National Survey of Personal Trust Accounts," (ABA mimeo., 1959) p. 4. For 1953, trust funds is the average of Goldsmith's year-end values for 1952 and 1953 for personal and common trust funds combined, Studies in the National Balance Sheet, Vol. II, pp. 122f

Column 5, Nonprofit institutions, was derived by applying to the mid-year asset values of nonfarm households (column 1) the percent that each asset held by nonprofit institutions in 1949 was of that asset held by households in 1949. See Goldsmith, A Study of Savings in the United States, Vol. III, p. 72 This ratio estimating procedure was made necessary because 1949 is the last year for which Goldsmith estimated a nonprofit sector. Goldsmith points out in the preface page to the 1949 nonprofit sector balance sheet, the estimates are rough approximations: "Whoever reads the notes to the tables -- or has worked in the field -- will be aware of how precarious the estimates are...", (*Ibid.* p. 449.). In spite of the roughness of the 1949 estimate, it is appropriate to use the estimate of the outstanding authority in the field as a basis to adjust downward the assets of the nonfarm household sector, which are known to be too high.

Column 6: To arrive at the total wealth concept for the individual sector, the assets and liabilities of farm households and unincorporated businesses were added to, and those of nonprofit institutions were subtracted from, the nonfarm household sector. (The assets and liabilities of trust funds are already included in the nonfarm household sector.) Thus Individuals total wealth is the sum of columns 1, 2, and 3, minus column 5.

Column 7: To obtain a prime wealth individual sector, assets of trust funds and pension reserves were subtracted from total wealth. Thus Individuals prime wealth is the sum of columns 1, 2, and 3, minus columns 4 and 5 and minus private and government pension and retirement funds.

Table 3. TOP WEALTH—HOLDERS SHARE IN NATIONAL BALANCE SHEET ACCOUNTS BY TYPE OF PROPERTY FOR 1958 AND 1953

Type of Property	1958			1953		
	Top wealth—holders *	Individuals national balance sheet prime wealth **	Share of top wealth—holders	Top wealth—holders *	Individuals national balance sheet prime wealth **	Share of top wealth—holders
	(Billion dollars)		(Percent)	(Billion dollars)		(Percent)
	(1)	(2)	(3)	(4)	(5)	(6)
Real estate	115.7	599.3	19.3	70.1	450.3	15.6
Federal bonds	16.3	60.2	27.1	17.4	57.6	30.2
State and local bonds	12.2	15.7	77.7	10.8	10.7	100.9
Other bonds	3.7	4.9	75.5	2.8	2.6	107.7
Corporate Stock	195.4	257.3	75.9	105.7	138.2	76.5
Cash	39.5	209.4	18.9	44.6	156.0	28.6
Notes and mortgages	17.9	43.9	40.8	10.5	32.1	32.7
Life insurance equity	13.3	103.0	12.9	++ 7.1	75.3	9.4
Miscellaneous assets +	57.9	268.6	21.6	39.6	205.6	19.3
Gross assets	472.0	1,562.3	30.2	309.2	1,128.4	27.4
Debt	42.9	222.8	19.3	27.7	133.8	20.7
Economic estate	429.0	1,339.5	32.0	281.5	994.6	28.3

Detail may not add to total because of rounding.

* For 1958, the value of all top wealth—holders wealth based upon decedents of known and unknown age with \$60,000 or more gross estate. For 1953, basic variant, a concept used by Lampman to include the value of all top wealth—holders wealth based upon decedents of known and unknown age for whom estate tax returns were filed. Conceptually these valuations were made in the same way except that Lampman included the value resulting from the blow-up of the wealth of 23 decedents with gross estates of less than \$60,000. The insignificance of this difference will be realized by noting that the total wealth of the living attributable to the 23 decedents with less than \$60,000 gross estate in Lampman's estimate is \$79,000,000 or .02 percent of his \$309,203,000,000 basic variant. Amounts shown in column 4 are from Lampman, op.cit., pp. 191f.

** Prime wealth estimates are from table 2. For the derivation of these estimates see the notes to that table.

+ Top wealth—holders "Miscellaneous assets" includes annuities and the national balance sheet estimates excludes Pension and retirement funds to achieve comparability with Lampman. The annuity figure for 1958, \$19.8 billion, (shown in talbe 1) was adjusted to \$2.7 billion before being included in miscellaneous assets. See footnote 21.

++ The value of life insurance equity shown by Lampman in table 90 (Lampman, op. cit. pp. 192f) appears to be incorrect. The correct value is believed to be at least \$8.7 billion. He shows \$8.7 billion in table 23, Ibid., p. 51, before an addition for the estate of decedents with age unspecified. Because, a negative value for insurance equity for the age unspecified group is not possible, we assume this figure to understate by at least 1.6 billion the basic variant value of life insurance equity of top wealth—holders. Based on a value of \$8.7 billion, the share of top wealth—holders in life insurance equity in 1953 would be 11.6 percent.

Table 4. SHARE OF TOP ONE PERCENT OF WEALTH—HOLDERS IN NATIONAL BALANCE SHEET ACCOUNTS MID—YEAR 1958 AND 1953

Type of Property	Top One Percent of Wealth—holders	
	1958 *	1953
	(Percent)	
Real estate	15.7	15.3
Federal bonds	22.8	30.2
State and local bonds	76.4	100.9
Other bonds	69.4	107.7
Corporate Stock	71.0	76.0
Cash	15.2	28.1
Notes and mortgages	35.1	31.5
Life insurance equity	11.0	9.3
Miscellaneous assets 3*	18.8	18.5
Gross assets	26.7	26.1
Debt	17.2	20.5
Economic estate	28.1	27.8
Economic estate		

* Based on asset holdings after adjustment for special case. See notes to table 3.

Table 5. SHARE OF TOP ONE PERCENT OF ADULTS IN NATIONAL BALANCE SHEET ACCOUNTS. SELECTED YEARS 1922 TO 1958

Year	Share of Top One Percent of Adult Wealth—holders
	(Percent)
1922	31.6
1929	36.3
1933	28.3
1939	30.6
1945	23.3
1949	20.8
1953	24.3
1954	24.0
1956	26.0
1958	23.8

NOTE: All figures except that for 1958 are from Lampman, op. cit. p. 204. The percent of basic variant wealth owned by the top one percent of adult wealth—holders for the years 1922 through 1956 was computed by Lampman on the basis of his balance sheets. The 1958 share is based on the balance sheet presented in this study. No adjustment has been made for Lampman's 1953 estimates which overstated the gross estate of the age unknown group and understated life insurance equities. See notes to table 3.

by top wealth-holders in perspective, their total number can be compared to the total population. On July 1, 1958, according to the Bureau of the Census, the population of the United States was 174.9 million. Top wealth-holders according to our estimate numbered 2.6 million. Therefore, in 1958 1.5 percent of the population held 32 percent of net prime wealth owned by all individuals in the United States.

For a comparison of the shares of top wealth-holders in 1953 and 1958, table 3 also shows Lampman's 1953 estimates as a percent of the mid-1953 balance sheet.²² The shares of real estate, notes and mortgages, and life insurance equity held by top wealth-holders appear to have increased since 1953. Federal bonds, corporate stock, and miscellaneous assets appear to represent about the same proportion of their respective totals in 1958 as they did in 1953. State and local bonds, other bonds, and cash represent lesser shares in 1958 than in 1953. The fact that large sampling variability may attach to individuals assets, particularly those with small aggregates such as municipal and "other" bonds which tend to be held by a very small number of persons, demands caution in interpreting differences between 1953 and 1958. It is unlikely that a large swing in the share of a particular asset owned by top wealth-holders occurred in a period as short as five years.

The most reliable estimates are those for gross and economic estate. Table 3 shows that top wealth-holders owned 27.4 percent of gross and 28.3 percent of net prime wealth in 1953, but increased their share to 30.2 and 32.0 percent respectively by 1958. These data support Lampman's conclusion that the share of top wealth-holders has been increasing since 1949.²³

The increased share of top wealth-holders in prime wealth between 1953 and 1958 is probably understated. Because of what appears to be an arithmetic error, Lampman included \$16.4 billion in gross estate as the blow-up of assets of decedents of unknown age.²⁴ The correct amount according to the procedure described by Lampman should be \$7.6 billion.²⁵ If \$7.6 billion is taken as the correct estimate of gross estate for the age

unknown group, then his estimate for all wealth-holders would be reduced to \$300.4 billion. On this basis, the share of top wealth-holders in national balance sheet gross prime wealth in 1953 would have been 26.6 percent compared to 30.2 percent in 1958.

But in order to get at the concentration of wealth, a constant percentage of the population should be considered. The proportion of the total population represented by top wealth-holders has been increasing. In 1953 the group consisted of approximately 1.66 million persons, or about 1.04 percent of the total U.S. population. In 1958 the number of persons had risen to 2.60 million or 1.48 percent of the total population.

Table 4 shows the percent of each asset in total prime wealth owned by the top 1 percent of all wealth-holders. In the aggregate, the wealthiest strata have at least maintained their share position of 1953. With respect to specific assets, the top 1 percent of wealth-holders held about the same share of real estate, corporate stock, notes and mortgages, life insurance equity, and miscellaneous assets in 1958 as in 1953. The share of all types of bonds in the hands of the top 1 percent of wealth-holders declined as did cash holdings.

A longer perspective of the concentration of wealth is available if one uses Lampman's data for the top 1 percent of adults. Table 5 shows the proportion of wealth held by the top 1 percent of persons over 20 years of age from 1922 to 1958. The estimate of 23.8 percent for 1958 is slightly lower than Lampman's estimates for 1953, 1954, and 1956. When it is remembered that there are differences in the balance sheet used for 1958 and for other years, that sampling errors exist in all the estimates, and that the wealth of the age unknown is overstated in 1953, one is forced to look at the whole series of estimates to assess changes in the concentration of wealth. Doing this, it appears that over the period of these estimates [wealth] was most highly concentrated in the 1920's, decreased in concentration during the depression and war years, and has been increasing since 1949. Lampman has already pointed out this movement in concentration, we merely add the most recent data in support of his observation.²⁶

The 1962 Internal Revenue Service Estimates

The first IRS estate multiplier study will be published in 1966.²⁷ Besides updating past estimates it will incorporate technical improvements in mortality rates and insurance valuation. Decedent of unknown age will be imputed the average age of all decedents of the same sex in the same size of gross estate class.

Data from the study "Social and Economic Differentials in Mortality Rates" being done by Hauser at the University of Chicago will be used to evaluate and probably adjust age-sex-specific mortality rates.

The adjustment for life insurance proceeds will be made on the basis of a special study being done for the IRS by the Institute of Life Insurance. A return filed for an estate which includes life insurance proceeds must provide a statement of the face value, policy loans, accumulated dividends and proceeds (among other things) for each life insurance contract on the life of the decedent. The information is supplied by attaching to the return a Form 712 completed by the carrier. Unfortunately, cash surrender value has not been one of the items of information requested. The Institute of Life Insurance, however, collected from a group of companies cash surrender value and age of decedent as well as all the information requested in Form 712 each time the form was completed during a recent two-month period. The Institute plans to tabulate ratios of cash surrender value to proceeds by age group. This set of ratios will be used to reduce the life insurance proceeds of decedents in each age group for the 1962 estimate. The decedents for whom the information is being tabulated are not, of course, those for whom estate tax returns were filed in 1963. Our assumption is that the ratios do not vary much from year to year.

The IRS study will cross-classify top wealth-holders' gross and net wealth by size, age, sex, marital status, State, and type of asset. Of particular interest to social scientists will be the distributions by net wealth after adjustment of life insurance to equity value. These distributions will permit better estimates of the concentration of wealth than has been possible with the "size of

gross estate" classification used in earlier estimates.

Future Application of the Estate Multiplier in the U.S.

In presenting the following ideas we wish to emphasize they represent the thinking of the authors and of scholars who have worked in the field, not the official position or plans of the IRS and Treasury.

The estate tax returns are an underdeveloped source of economic information. True, successive applications of the estate multiplier have expanded the number of questions asked of the returns, but their full potential is far from exploited.

What is the propensity of top wealth-holders for financial risk? Does increased wealth shift portfolio composition toward growth stocks? How significant is the closely held corporation? Since estate tax returns provide security information on an issue-by-issue basis, indices could be devised to provide answers to these questions.²⁸

What are the occupational characteristics of top wealth-holders? Occupation and social security number of the decedent are contained in the return. A cross-sectional occupation pattern may be obtainable directly from the returns; the life cycle pattern might be obtained by a social security number match with SSA work history records. Further, wealth-holding by occupational groups could be estimated.²⁹

What is the income of top wealth-holders? What is the income of the heirs? Using social security numbers, estate tax returns can be matched with previous income tax returns, thus putting together the income flow and wealth stock. A slight modification of the estate tax returns would make possible a study of the intergenerational flows by wealth and income of decedents, and income of heirs.

What is the relation between *inter vivos* giving and transmission of assets at death? What are the effects of the creation of *inter vivos* and testamentary trusts? The addition of grantors' and

donors' social security numbers on fiduciary and gift tax returns would permit a computer match of these returns with estate tax returns.

For further applications of the estate multiplier to U.S. data certain methodological improvements constitute the immediate needs. The tabulation of returns for four or five consecutive years, sorted by year of death, would minimize the error caused by the assumption of inferred year of death. It would also permit a reduction of sampling variability by combining returns for two or more years.

Reporting by life insurance companies of cash surrender value for each contract in the estates of decedents for which an estate tax return is filed would vastly improve estimates of life insurance equity.

Further research to improve mortality rates assigned to decedents filing estate tax returns should be undertaken. One path is that of relating death to income size. Another would be an exploration of the information on the cause of death (supplied by a physician) and occupation contained in the return.

FOOTNOTES

1. See Dorothy S. Projector, "Survey of Financial Characteristics of Consumers," Federal Reserve Bulletin, March 1964, p. 289.
2. See Constantine A. Yeracaris, "Differential Mortality, General and Cause Specific in Buffalo, 1934-41," Journal of the American Statistical Association, December 1955; Louis I. Dublin, Alfred J. Lotka, and Mortimer Spiegelman, Length of Life: A Study of the Life Table, New York: The Ronald Press, 1949; I.M. Moriyama and L. Guralnick "Occupational and Social Class Differences in Mortality" in Trends and Differentials in Mortality, proceedings of the 1955 Annual Conference, Milbank Memorial Fund. Unpublished statistics provided by the Department of Health, City of New York, show mortality rates for high income health districts to be significantly lower than those for low income districts.
3. C. Lowell Harris, "Wealth Estimates as Affected by Audit of Estate Tax Returns," National Tax Journal, December 1949, p. 333.
4. The interval devolution method estimates total wealth by multiplying the wealth of decedents by the average interval, in years, between generations. A discussion of attempt to use the interval devolution method will be found in G.H. Knibbs, The Private Wealth of Australia and its Growth, Melbourne: McCarron, Bird and Co., 1918, pp. 168-81.
5. Bernard Mallet, "A Method of Estimating Capital Wealth from Estate Duty Statistics," Journal of the Royal Statistical Society, March 1908, pp. 65-84.
6. Victorian Yearbook, 1911-12, p. 216.
7. Knibbs, op. cit.
8. G.W. Daniels and H. Campion, The Distribution of National Capital, Manchester: Manchester University Press, 1936; and H. Campion, Public and Private Property, Oxford: Oxford University Press, 1939.
9. Kathleen Langly, "The Distribution of Capital in Private Hands in 1936-1938 and 1946-1947," Bulletin of the Oxford University Institute of Statistics, December 1950, p. 339 and February 1951, p. 33; and Kathleen Langly, "The Distribution of Private Capital, 1950-51," Bulletin of the Oxford University Institute of Statistics, January 1954, p. 1.
10. A.M. Cartter, "A New Method of Relating British Capital Ownership and Estate Duty Liability to Income Groups" Economica

- August 1953, p. 247.
11. H.F. Lydall and D.G. Tipping, "The Distribution of Personal Wealth in Britain," Bulletin of the Oxford University Institute of Statistics, January 1961, p. 96.
 12. J.R.S. Revell "Assets and Age," Bulletin of the Oxford University Institute of Statistics, March 1962, p. 363. Revell further analyzes the 1957 and 1958 British data in his forthcoming monograph on national balance sheets, now in manuscript.
 13. Report of the Commissioners of Her Majesty's Inland Revenue for the Year Ended 31st March 1961, London: Her Majesty's Stationary Office, 1962, pp. 154-160.
 14. Horst Mendershausen, "The Pattern of Estate Tax Wealth," in Raymond W. Goldsmith, Dorothy S. Brady, and Horst Mendershausen, A Study of Savings in the United States, Vol. III Princeton: Princeton University Press, 1956.
 15. Ibid., p. 299.
 16. Robert J. Lampman, The Share of Top Wealth-holders in National Wealth 1922-56, Princeton: Princeton University Press, 1962.
 17. Moriyama and Guralnick, op. cit.
 18. James D. Smith, unpublished research submitted to the University of Oklahoma in connection with a doctoral dissertation. The original estate multiplier estimate by Smith was based upon tabulations in Statistics of Income--1958, Fiduciary, Gift, and Estate Tax Returns, and the composition of wealth by asset type was estimated by a different technique. However, Jeannette Fitzwilliams and Raymond W. Goldsmith generously made available a special Internal Revenue Service tabulation (upon which considerable work toward an estate multiplier estimate had been made by Fitzwilliams) which permitted the direct estate multiplier estimate of asset composition in table 1.
 19. Raymond W. Goldsmith, Robert E. Lipsey and Morris Mendelson, Studies in the National Balance Sheet.
 20. Lampman, op. cit., pp. 191-5.
 21. The occurrence of such a high value may merely reflect the excessive sampling variability attaching to a rare event, or it may have been the result of faulty transcription in processing the returns by the Internal Revenue Service. An attempt to check the Internal Revenue transcription was frustrated by the fact that the edit sheets and punch cards used in the tabulation of the 1958 returns had been destroyed.
 22. The major differences in the 1953 Lampman balance sheet and that constructed for this study are: 1. Lampman used end-of-year values. 2. Lampman apparently allocated assets to trust funds on the basis of the findings reported in Goldsmith and Shapiro's "Estimates of Bank-Administered Trust Funds," Journal of Finance, March 1959, pp. 11-17; the mid-1953 balance sheet uses Goldsmith's later estimates in Studies in the National Balance Sheet and the "Report of Survey of Trust Accounts." 3. Lampman's estimate of 16.1 billion "equities in mutual financial institutions" held by the household sector appears far too large in comparison with Goldsmith's National Balance Sheet entry of \$4.8 billion.
 23. Lampman, op. cit., p. 24.
 24. Lampman, op. cit., p. 62.
 25. See Lampman, op. cit., p. 55. In discussion with Lampman he has stated to Smith that he was unable to reconcile the \$16.4 billion shown in his table 29 with the procedure described on page 55 of The Share of Top Wealth-Holders, but that the procedure should yield the correct estimate for the age unknown group.
 26. Lampman, op. cit., p. 24.

27. Yeoman labor in preparing the specifications for the IRS estimate was done by Jeannette Fitzwilliams. She also, with Robert Lampman and Raymond Goldsmith, contributed to the initial planning of the project, at the request of the late Ernest Engquist, Director of the Statistics Division. Helen Demond with assistance from Jeannette Fitzwilliams drafted specifications for the project.
28. See, for instance, I. R. Atkinson, The Pattern of Financial Asset Ownership: Wisconsin Individuals, 1949, Princeton: Princeton University Press, 1956.
29. See Leonard Hamilton, "The Distribution of Capital Among the Medical Profession in England and Wales, 1940-41," Bulletin of the Oxford University Institute of Statistics, January 1950, p.1.

Technical Appendix*

The estate multiplier technique rests on the assumption that death draws a random sample, stratified by age and sex, of the living population.

If one has available age-sex-specific mortality rates, an estimate of total wealth can be derived as:

$$W = \sum_{i=1}^m \sum_{j=1}^2 \frac{V_{ij}}{M_{ij}} w_{ij}$$

where w_{ij} is the wealth of decedents, V_{ij} is the number of living persons, and M_{ij} is the number of deaths, all associated with the i th age and j th sex. Decedents' wealth can be dimensioned to any degree, subject only to the tolerable sampling error for the purposes to which the estimates are to be put. For instance, w_{ijk} may be used to represent the wealth of decedents of the i th age, j th sex, of asset type k in the g th gross wealth size class. Using such a four-way classification, there is a partitioned matrix W , such that the distribution of wealth by any combination of the classification categories can be obtained as a result of pre-and/or post-multiplication of W by suitable vectors and/or matrices. The matrix W is defined by $W = (t_{ij} w_{ijk})$ where $t_{ij} = V_{ij} / M_{ij}$ and where $i = 1, 2, \dots, m$; $j = 1, 2$; $k = 1, 2, \dots, n$; $g = 1, 2, \dots, s$.

If we let $W_{kg} = \begin{pmatrix} t_{11}W_{11kg} & t_{12}W_{12kg} \\ \cdot & \cdot \\ \cdot & \cdot \\ t_{m1}W_{m1kg} & t_{m2}W_{m2kg} \end{pmatrix}$ Then we can write: $W = \begin{pmatrix} W_{11} & W_{12} & \cdot & \cdot & \cdot & W_{1s} \\ W_{21} & W_{22} & \cdot & \cdot & \cdot & W_{2s} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ W_{n1} & W_{n2} & \cdot & \cdot & \cdot & W_{ns} \end{pmatrix}$

The following example illustrates how a particular cross-category distribution of wealth may be obtained from W .

For any integer p , let J_p be the p -dimensional column vector of ones. Then, multiplication of W on the left by J'_m gives a $1 \times 2s$ matrix whose elements are column sums of W :

$$\begin{aligned} J'_m W &= [(J'_m W_{11} + J'_m W_{21} + \dots + J'_m W_{n1}), (J'_m W_{12} + J'_m W_{22} + \dots + J'_m W_{n2}), \dots, (J'_m W_{1s} + J'_m W_{2s} + \dots + J'_m W_{ns})] \\ &= ([\sum_{k=1}^n J_m W_{k1}], [\sum_{k=1}^n J_m W_{k2}], \dots, [\sum_{k=1}^n J_m W_{ks}]) \end{aligned}$$

* The assistance of Michael G. Billings, Mathematical Statistician, Statistics Division, Internal Revenue Service, is gratefully acknowledged.

$$\text{Now, } J'_m W_{kg} = (1, 1, 1, \dots, 1) \begin{pmatrix} t_{11} W_{11kg} & t_{12} W_{12kg} \\ t_{21} W_{21kg} & t_{22} W_{22kg} \\ \cdot & \cdot \\ \cdot & \cdot \\ t_{m1} W_{m1kg} & t_{m2} W_{m2kg} \end{pmatrix} = \left(\sum_{i=1}^m t_{i1} W_{i1kg}, \sum_{i=1}^m t_{i2} W_{i2kg} \right)$$

Therefore $J'_{mn} W =$

$$\left(\left[\sum_{k=1}^n \sum_{i=1}^m t_{i1} W_{i1k1}, \sum_{k=1}^n \sum_{i=1}^m t_{i2} W_{i2k1} \right], \left[\sum_{k=1}^n \sum_{i=1}^m t_{i1} W_{i1k2}, \sum_{k=1}^n \sum_{i=1}^m t_{i2} W_{i2k2} \right], \dots, \left[\sum_{k=1}^n \sum_{i=1}^m t_{i1} W_{i1ks}, \sum_{k=1}^n \sum_{i=1}^m t_{i2} W_{i2ks} \right] \right)$$

Each of the s 1×2 vectors in $J'_{mn} W$ has as one of its elements the total wealth of females and the other of males in a given wealth class. Thus, the product matrix $J'_{mn} W$ can be thought of as the distribution of total wealth by size of wealth-holding and sex of holder.

Similarly, the distribution of wealth by size of holding, sex and age of holder can be obtained as follows: Let $I_{pq} = (I_p, I_p, I_p, \dots, I_p)$ be the p -rowed matrix consisting of q $p \times p$ identity matrices.

Then, $I_{mn} W = \left(\sum_{k=1}^n W_{k1}, \sum_{k=1}^n W_{k2}, \dots, \sum_{k=1}^n W_{ks} \right)$. The submatrix $\sum_{k=1}^n W_{kg}$ of $I_{mn} W$ has as its ij th component the sum of the wealth size class g held by persons in age group i , sex group j .

The composition of wealth by asset type, age and sex of holder is given by $WI'_{2s} = \left(\sum_{g=1}^s W_{1g}, \sum_{g=1}^s W_{2g}, \dots, \sum_{g=1}^s W_{ng} \right)'$. The submatrix

$\sum_{g=1}^s W_{kg}$ of WI'_{2s} has as its ij th component the value of asset type k held by persons of age group i , sex group j .

Once the matrix W has been set-up, any desired distribution of wealth using the selected variables can be extracted by choosing the appropriate vector and/or matrix multipliers. The following definitions and table summarize the multipliers which would be used to obtain the various breakdowns in our four-way classification.

Definitions of the multipliers:

1. For any integer p , J_p is the column vector of p ones.
2. For any integers p and q , I_{pq} is the p -rowed matrix which can be partitioned into q $p \times p$ identity matrices:

$$I_{pq} = (I_p, I_p, I_p, \dots, I_p)$$

3. For any integers $m, n, p, p \leq n$, J_{nm}^p is the $n \times m$ matrix whose p th row is J'_m , where J_m is defined in (1).
4. For any integers m, n , K_{nm} is the $n \times nm$ matrix which can be partitioned as follows:

$$K_{nm} = (J_{nm}^1, J_{nm}^2, J_{nm}^3, \dots, J_{nm}^n), \text{ where } J_{nm}^p \text{ is defined in (3).}$$

With the matrices, vectors, and their transposes, defined above, any desire cross-classified wealth distribution can be obtained from W . If we think of W as an hierarchy of classification, we can write $W = \text{asset (age-sex) size}$. Then,

1. Multiplication of W on the left by I_{mn} eliminates an asset.
2. Multiplication of W on the right by I'_{2s} eliminates size.
3. Multiplication of W on the left by K_{nm} eliminates age.
4. Multiplication of W on the right by K'_{s2} eliminates sex.
5. Multiplication of W on the left by J'_{mn} eliminates asset and age.
6. Multiplication of W on the right by J_{2s} eliminates size and sex.
7. Sequential eliminations are obtained by performing operations in sequence (see table below).

MATRIX MULTIPLIERS, OPERATIONS AND PRODUCTS

<u>Multipliers</u>		Eliminates	Product	<u>Product gives</u> wealth-holding by
Left	Right			
J'_{mn}	-	Asset, age	$J'_{mn}W$	Sex, size
-	J_{2s}	Sex, size	WJ_{2s}	Asset, age
I_{mn}	-	Asset	$I_{mn}W$	Age, sex, size
-	I'_{2s}	Size	WI'_{2s}	Asset, age, sex
K_{nm}	-	Age	$K_{nm}W$	Asset, sex, size
-	K'_{s2}	Sex	WK'_{s2}	Asset, age, size
I_{mn}	K'_{s2}	Asset, sex	$I_{mn}WK'_{s2}$	Age, size
K_{nm}	I'_{2s}	Age, size	$K_{nm}WI'_{2s}$	Asset, sex
I_{mn}	I'_{2s}	Asset, size	$I_{mn}WI'_{2s}$	Age, sex
I_{mn}	J_{2s}	Asset, size, sex	$I_{mn}WJ_{2s}$	Age
J'_{mn}	I'_{2s}	Asset, age, size	$J'_{mn}WI'_{2s}$	Sex
J'_{mn}	K'_{s2}	Asset, age, sex	$J'_{mn}WK'_{s2}$	Size
K_{nm}	J_{2s}	Age, sex, size	$K_{nm}WJ_{2s}$	Asset
K_{nm}	K'_{s2}	Age, sex	$K_{nm}WK'_{s2}$	Asset, size

