

AMERICAN SOCIETY OF PENSION PROFESSIONALS & ACTUARIES
JOINT BOARD FOR THE ENROLLMENT OF ACTUARIES
SOCIETY OF ACTUARIES

Enrolled Actuaries Basic Examination

EA-1

Date: Thursday, May 11, 2023

INSTRUCTIONS TO CANDIDATES

1. Special conditions generally applicable to all questions on this examination are found on the next page.
2. On this examination the symbol “ ” will be used to represent an annuity. On this examination the symbol “ ℓ ” will be used to represent the number of lives at age x .
3. This examination consists of 31 multiple-choice questions worth a total of 100 points. The point value for each question is shown in parentheses at the beginning of the question.
4. Your score will be based on the point values of questions that you answer correctly. No credit will be given for omitted answers and no credit will be lost for wrong answers; hence, you should answer all questions even those for which you have to guess.
5. Do not spend too much time on any one question. If a question seems too difficult, leave it and go on.
6. While every attempt is made to avoid defective questions, sometimes they do occur. If you believe a question is defective, the supervisor or proctor cannot give you any guidance beyond the instructions on the computer screen.
7. Use the scratch paper booklets provided by Prometric for your scratch work. Extra scratch paper booklets are available if you run out of scratch paper in the booklet provided to you.

Exam EA-1

Answer Key EA-1 Spring 2023
March 1, 2023

Question	Answer	Points
1	D	4
2	C	3
3	E	4
4	E	4
5	B	3
6	D	4
7	E	3
8	D	3
9	E	4
10	D	2
11	B	3
12	A	2
13	B	2
14	A	3
15	B	3
16	B	4
17	A	3
18	C	3
19	C	4
20	E	2
21	D	3
22	B	4
23	C	3
24	B	4
25	A	4
26	B	3
27	D	4
28	C	3
29	D	2
30	A	4
31	D	3

CONDITIONS GENERALLY APPLICABLE TO ALL EA-1 EXAMINATION QUESTIONS

If applicable, the following conditions should be considered a part of the data for each question, unless otherwise stated or implied.

- (1) The normal retirement age is 65.
- (2) Retirement pensions commence at normal retirement age and are paid monthly for life at the beginning of each month.
- (3) There are no pre-retirement death or disability benefits.
- (4) Actuarial equivalence is based on the mortality table and interest rate assumed for funding purposes.
- (5) Interest rates that are compounded more frequently than annually are expressed as nominal rates.
- (6) Where multiple lives are involved, future lifetimes are assumed to be independent of each other.
- (7) The term “gross single premium” is equivalent to “contract single premium;” the term “net single premium” is equivalent to “single benefit premium;” the term “gross annual premium” is equivalent to “annual contract premium;” the term “net annual premium” is equivalent to “annual benefit premium.”
- (8) There are no policy loans in effect.
- (9) For a bond, the face amount and the redemption value are the same.
- (10) Interest rate equals yield rate.
- (11) The term “duration” means “Macaulay duration”.

If applicable, the preceding conditions should be considered a part of the data for each question, unless otherwise stated or implied.

Data for Question 1 (4 points)

Data for two funds:

	<u>Fund A</u>	<u>Fund B</u>
Interest/discount rate for first 10 years	$i^{(4)}=6.0\%$	$d^{(12)}=9.0\%$
Discount/interest rate for second 10 years	$d^{(4)}=9.0\%$	$i^{(12)}=12.0\%$
Initial amount in fund	<i>W</i>	<i>X</i>
Amount in fund at end of 20 years	<i>Y</i>	<i>Z</i>

There are no contributions to or withdrawals from either fund.

$$W + X = \$10,000$$

$$Y + Z = \$57,186$$

Question 1

In what range is ***Y***?

- (A) Less than \$27,500
- (B) \$27,500 but less than \$28,400
- (C) \$28,400 but less than \$29,300
- (D) \$29,300 but less than \$30,200
- (E) \$30,200 or more

Data for Question 2 (3 points)

An annuity-immediate pays \$2,400 per year for 20 years:

For the first 10 years, the annuity is payable annually.

Interest for this annuity is computed at 4.0% per year, compounded annually.

For the second 10 years, the annuity is payable semiannually.

Interest for this annuity is computed at 4.0% nominal, compounded quarterly during the second 10 years.

X = the present value of this annuity.

Question 2

In what range is X ?

- (A) Less than \$32,650
- (B) \$32,650 but less than \$32,700
- (C) \$32,700 but less than \$32,750
- (D) \$32,750 but less than \$32,800
- (E) \$32,800 or more

Data for Question 3 (4 points)

An annuity-immediate provides 15 annual payments:

Initial payment	\$5.00
Subsequent payments	Payments increase by \$2.00 per year until the payment reaches \$19.00. Payments then decrease by \$2.00 per year until the payment reaches \$5.00.

Interest rate: 5.0% per year, compounded annually

X = the present value of these payments.

Question 3

In what range is X ?

- (A) Less than \$100
- (B) \$100 but less than \$105
- (C) \$105 but less than \$110
- (D) \$110 but less than \$115
- (E) \$115 or more

Data for Question 4 (4 points)

Provisions of a loan:

Number of payments	10
Amount of each payment	\$5,000
Timing of payments	Annual payments commencing at the end of the first year
Interest rate	8.0% per year, compounded annually

Immediately after the sixth payment is made, an additional \$10,000 payment is made.

The loan is then re-amortized over a longer term to provide for annual payments of \$1,000 and a final smaller payment of X paid one year after the last \$1,000 payment. The interest rate for this re-amortization remains at 8.0% per year, compounded annually.

Question 4

In what range is X ?

- (A) Less than \$350
- (B) \$350 but less than \$450
- (C) \$450 but less than \$550
- (D) \$550 but less than \$650
- (E) \$650 or more

Data for Question 5 (3 points)

Smith borrows \$1,000 from Jones.

The terms of the loan are:

Repayment period	25 years
Payments	Level annual payments, made at end of year
Interest	4.0% per year, compounded annually

When Jones receives each repayment from Smith, Jones immediately reinvests the repayment at a nominal interest rate of 5.0% per year, compounded semiannually.

X = the equivalent level annual yield that Jones earned over the 25 years.

Question 5

In what range is X ?

- (A) Less than 4.59%
- (B) 4.59% but less than 4.61%
- (C) 4.61% but less than 4.63%
- (D) 4.63% but less than 4.65%
- (E) 4.65% or more

Data for Question 6 (4 points)

Smith purchases a 20-year bond that has the following terms:

Par value	\$1,000
Coupon rate	6.0% per year, payable semiannually
Yield rate	5.0% per year, compounded annually.

When Smith receives each coupon, Smith immediately reinvests it at a rate of interest of 6.0% per year, compounded quarterly.

X = Smith's effective annual rate of return over the term of the bond.

Question 6

In what range is X ?

- (A) Less than 5.20%
- (B) 5.20% but less than 5.30%
- (C) 5.30% but less than 5.40%
- (D) 5.40% but less than 5.50%
- (E) 5.50% or more

Data for Question 7 (3 points)

Pension fund asset information for a given year:

<u>Date</u>	<u>Asset value</u>	<u>Withdrawal</u>
1/1	\$1,500,000	--
3/1	--	\$100,000
9/1	--	\$100,000
12/31	\$1,650,000	--

There was also a \$250,000 deposit made to the fund during the year.

The dollar-weighted rate of return for the year was 6.45%.

X = the fraction of the year that the deposit was invested in the pension fund.

Question 7

In what range is X ?

- (A) Less than 0.33
- (B) 0.33 but less than 0.43
- (C) 0.43 but less than 0.53
- (D) 0.53 but less than 0.63
- (E) 0.63 or more

Data for Question 8 (3 points)

A bond is issued with the following provisions:

Face amount	\$10,000
Term	4 years
Coupons	3%, payable annually

The term structure of interest rates is as follows:

<u>Year</u>	<u>Spot rate</u>
1	2.5%
2	2.7%
3	3.0%
4	3.3%

X = the bond's price at issue.

Question 8

In what range is X ?

- (A) Less than \$9,775
- (B) \$9,775 but less than \$9,825
- (C) \$9,825 but less than \$9,875
- (D) \$9,875 but less than \$9,925
- (E) \$9,925 or more

Data for Question 9 (4 points)

Smith has an obligation (liability) to make level payments at the end of each year for 10 years beginning one year from today.

Smith decides to purchase a 3-year zero-coupon bond with a redemption value of \$700 and a 6-year zero-coupon bond with a redemption value of \$ X .

With the purchase of these zero-coupon bonds, Smith is in an immunized position.

The yield rate on Smith's asset/liability portfolio is an annual effective rate of 7.0%.

Question 9

In what range is X ?

- (A) Less than 1,400
- (B) 1,400 but less than 1,450
- (C) 1,450 but less than 1,500
- (D) 1,500 but less than 1,550
- (E) 1,550 or more

Data for Question 10 (2 points)

With regard to structuring assets and liabilities in a manner designed to eliminate the adverse effects of changes in the level of interest rates (i.e., full immunization), consider the following statements:

- I. The present value of cash inflow from the assets is equal to the present value of cash outflow from the liabilities.
- II. The duration of the assets is equal to the duration of the liabilities.
- III. The full immunization technique is designed to work for either large or small changes in interest rates.

Question 10

Which, if any, of the above statement(s) is (are) true?

- (A) I and II only
- (B) I and III only
- (C) II and III only
- (D) I, II, and III
- (E) The correct answer is not given by (A), (B), (C), or (D)

Data for Question 11 (3 points)

A fund is established and invested as follows:

	<u>Number of shares</u>	<u>Price per share</u>
Preferred stock	10,000	\$50
Common stock	5,000	\$100

At the end of the first year, all shares of the common stock are sold for \$118 per share, with proceeds remaining in the fund.

After the common stock is sold, the value of the fund is \$1,075,000.

Any dividends are withdrawn from the fund as they are earned.

X = the unrealized gain (loss) on the fund for the first year.

Question 11

In what range is X ?

- (A) Less than (\$30,000)
- (B) (\$30,000) but less than (\$10,000)
- (C) (\$10,000) but less than \$10,000
- (D) \$10,000 but less than \$30,000
- (E) \$30,000 or more

Data for Question 12 (2 points)

You are given the following information for a fund for the year:

<u>Date</u>	<u>Account Balance</u>	<u>Deposit</u>
1/1	\$500,000	
8/1		\$100,000

The dollar-weighted rate of return equals 18.46%.

Realized gain equals \$20,000.

Interest plus dividends equals \$10,000.

There are no other cash flows during the year.

X = the unrealized gain during the year.

Question 12

In what range is X ?

- (A) Less than \$75,000
- (B) \$75,000 but less than \$85,000
- (C) \$85,000 but less than \$95,000
- (D) \$95,000 but less than \$105,000
- (E) \$105,000 or more

Data for Question 13 (2 points)

On 1/1, a company purchases one-year corporate bonds with expected proceeds of \$2,000,000 on 12/31 of the same year.

The bonds have the following terms:

Each bond is redeemable at par value of \$1,000.

Coupon rate: 8.0% per year, payable end-of-year

The probability of default during the year on each bond is 6.0%.

If a bond defaults, the recovery value at year-end is \$600.

The default of any bond is independent of the default of any other bond.

X = the total number of bonds to be purchased.

Question 13

In what range is X ?

- (A) Fewer than 1,900
- (B) 1,900 but fewer than 1,905
- (C) 1,905 but fewer than 1,910
- (D) 1,910 but fewer than 1,915
- (E) 1,915 or more

Data for Question 14 (3 points)

Smith (age 40) begins making deposits of 6.0% of each year's annual salary into a fund at the beginning of each calendar year.

Data for Smith:

Annual salary for first deposit: \$45,000

Anticipated salary increases: 4.0% per year, compounded annually.

The fund earns 5.0% per year, compounded annually.

X = expected value of Smith's fund at age 60 before the deposit at age 60.

Question 14

In what range is X ?

- (A) Less than \$132,500
- (B) \$132,500 but less than \$134,000
- (C) \$134,000 but less than \$135,500
- (D) \$135,500 but less than \$137,000
- (E) \$137,000 or more

Data for Question 15 (3 points)

A plan uses 4-year select and ultimate termination rates:

Select rates:

<u>Years of service</u>	<u>Rate</u>
0	15.0%
1	14.0%
2	12.0%
3	10.0%

Ultimate rates:

<u>x</u>	<u>Rate</u>	<u>x</u>	<u>Rate</u>
25	15.0%	31	6.0%
26	15.0%	32	5.5%
27	12.0%	33	5.0%
28	9.0%	34	4.5%
29	8.0%	35	4.0%
30	7.5%		

There are no decrements other than termination.

Smith was hired at age 27.

X = the probability that Smith is still an active participant at age 33.

Question 15

In what range is X ?

- (A) Less than 50%
- (B) 50% but less than 55%
- (C) 55% but less than 60%
- (D) 60% but less than 65%
- (E) 65% or more

Data for Question 16 (4 points)

Smith (age 60) is to receive a 4-year annuity-immediate with payments of \$20,000 per year, payable annually.

Mortality is calculated based on the following:

Basic mortality $l_x^{Basic} = 100 - x, x \geq 60$

Projected mortality $q_x^{Projected} = q_x^{Basic} (0.97)^t, t = 0, 1, 2, \dots$

Interest rate: 0.0%

X = the single premium for Smith's annuity.

Question 16

In what range is X ?

- (A) Less than \$75,100
- (B) \$75,100 but less than \$76,500
- (C) \$76,500 but less than \$77,900
- (D) \$77,900 but less than \$79,300
- (E) \$79,300 or more

Data for Question 17 (3 points)

The rates of mortality for impaired lives are 150% of the rates of standard lives.

The following mortality rates are for standard lives:

\underline{x}	\underline{q}_x
95	3/10
96	4/10
97	5/10
98	2/3

X = the curtate life expectancy for an impaired life age 95.

Question 17

In what range is X ?

- (A) Less than 0.85
- (B) 0.85 but less than 1.00
- (C) 1.00 but less than 1.15
- (D) 1.15 but less than 1.30
- (E) 1.30 or more

Data for Question 18 (3 points)

Selected commutation functions:

\underline{x}	\underline{D}_x
40	8,523
41	7,982
42	7,473

$$\ddot{a}_{42} = 14.186$$

Interest rate: 5.0% per year, compounded annually.

$$X = 100,000A_{40}$$

Question 18

In what range is X ?

- (A) Less than \$30,750
- (B) \$30,750 but less than \$31,250
- (C) \$31,250 but less than \$31,750
- (D) \$31,750 but less than \$32,250
- (E) \$32,250 or more

Data for Question 19 (4 points)

Smith (age 40) purchases an annuity with the following provisions:

If Smith survives to age 65, an annuity of \$10,000 per year will be paid to Smith commencing at age 65 and continuing for Smith's lifetime

If Smith dies before age 65, the single premium will be refunded without interest when Smith would have attained age 65

The annuity contract specifies the following commutation functions at 4.0% interest, compounded annually:

\underline{x}	N_x
40	1,320
41	1,250
...	...
65	230
66	209

X = the net single premium for this annuity.

Question 19

In what range is X ?

- (A) Less than \$34,000
- (B) \$34,000 but less than \$35,000
- (C) \$35,000 but less than \$36,000
- (D) \$36,000 but less than \$37,000
- (E) \$37,000 or more

Data for Question 20 (2 points)

Selected survival probabilities:

\underline{x}	\underline{p}_x
55-59	0.99
60-64	0.95
65	0.90

Interest rate: 7.0% per year, compounded annually

$$a_{65} = 8.194$$

$$X = {}_{10|}\ddot{a}_{55}$$

Question 20

In what range is X ?

- (A) Less than 3.1
- (B) 3.1 but less than 3.2
- (C) 3.2 but less than 3.3
- (D) 3.3 but less than 3.4
- (E) 3.4 or more

Data for Question 21 (3 points)

Smith (age 45) purchases a single premium annuity with the following characteristics:

Single premium	\$100,000
Monthly payment	X at the beginning of each month
Payment period	For Smith's lifetime, with payments guaranteed for the first 120 months
Interest rate	3.0% per year, compounded annually

Selected commutation functions:

$$D_{45} = 2,392,905$$

$$D_{55} = 1,639,330$$

$$N_{55} = 24,032,177$$

Question 21

In what range is X ?

- (A) Less than \$440
- (B) \$440 but less than \$445
- (C) \$445 but less than \$450
- (D) \$450 but less than \$455
- (E) \$455 or more

Data for Question 22 (4 points)

Terms of an annuity issued to a life age 71:

Frequency of payments	Annually at the beginning of each year
Payment amount	Account balance divided by the annuitant's curtate expectation of life on the payment date
Account balance at issue date	\$100,000
Assumed rate of return	6.0% per year, compounded annually

Selected values:

$\frac{x}{71}$	$\frac{p_x}{0.9477}$	$\frac{e_x}{9.30}$
72	0.9438	
73	0.9394	

X = the expected value of the third payment.

Question 22

In what range is X ?

- (A) Less than \$10,650
- (B) \$10,650 but less than \$10,700
- (C) \$10,700 but less than \$10,750
- (D) \$10,750 but less than \$10,800
- (E) \$10,800 or more

Data for Question 23 (3 points)

Selected probabilities for three independent lives:

$$q_x = 0.014964$$

$$q_y = 0.024765$$

$$q_z = 0.043316$$

X = the probability that exactly two of the three lives survive for one year.

Question 23

In what range is X ?

- (A) Less than 0.0534
- (B) 0.0534 but less than 0.0785
- (C) 0.0785 but less than 0.1036
- (D) 0.1036 but less than 0.1287
- (E) 0.1287 or more

Data for Question 24 (4 points)

Selected probabilities of survival:

$${}_{20}p_{30} = 0.75$$

$${}_{25}p_{50} = 0.55$$

$${}_{20}p_{55} = 0.60$$

X = the probability that two independent lives, age 30 and age 50, will both die between ages 55 and 75.

Question 24

In what range is X ?

- (A) Less than 0.095
- (B) 0.095 but less than 0.105
- (C) 0.105 but less than 0.115
- (D) 0.115 but less than 0.125
- (E) 0.125 or more

Data for Question 25 (4 points)

A multiple-decrement table has two sources of decrement: death (d) and retirement (r):

Deaths are assumed to be uniformly distributed throughout the year.

30% of retirements are assumed to occur on 4/1 and
70% are assumed to occur on 7/1 of each year.

Selected values:

$$q_x^{(d)} = 0.05$$

$$q_x^{(r)} = 0.14$$

$$X = q_x^{(d)}$$

Question 25

In what range is X ?

- (A) Less than 0.048
- (B) 0.048 but less than 0.050
- (C) 0.050 but less than 0.052
- (D) 0.052 but less than 0.054
- (E) 0.054 or more

Data for Question 26 (3 points)

A multiple-decrement table has two sources of decrement: death (d) and withdrawal (w).

$$\ell_x^{(\tau)} = 1,000$$

$$q_x^{(d)} = 0.20$$

$$q_x^{(w)} = 0.40$$

Each decrement is uniformly distributed over the year of age in the associated single-decrement tables.

X = the number expected to survive to age $(x+1)$ in the multiple-decrement table.

Question 26

In what range is X ?

- (A) Less than 445
- (B) 445 but less than 452
- (C) 452 but less than 459
- (D) 459 but less than 466
- (E) 466 or more

Data for Question 27 (4 points)

Under a pension plan, Smith (age 57) is entitled to a monthly life annuity benefit of \$1,000 payable at the beginning of each month, commencing immediately.

Smith elects a Social Security level income option that is actuarially equivalent to this life annuity. Under this option, Smith's total monthly income from the pension plan plus the amount Smith will receive from Social Security remains level for Smith's lifetime.

Smith's monthly Social Security benefit will be \$1,500 commencing at age 62.

Selected actuarial values:

$$\ddot{a}_{57}^{(12)} = 12.431$$

$$\ddot{a}_{62}^{(12)} = 11.678$$

$${}_5E_{57} = 0.692$$

X = the monthly pension benefit payable to Smith from the pension plan from age 57 to age 62 under the Social Security level income option.

Question 27

In what range is X ?

- (A) Less than \$1,700
- (B) \$1,700 but less than \$1,800
- (C) \$1,800 but less than \$1,900
- (D) \$1,900 but less than \$2,000
- (E) \$2,000 or more

Data for Question 28 (3 points)

A pension plan provides the following:

Normal retirement benefit	\$10,000, payable annually as a life annuity commencing at normal retirement age
Normal retirement age	62

The pension plan offers an early retirement benefit that is actuarially equivalent to the normal retirement benefit.

The pension plan also offers an optional form of benefit of a life annuity with 5 years certain that is actuarially equivalent to a single life annuity.

Selected values from a life table:

\underline{x}	\underline{N}_x	\underline{x}	\underline{N}_x
55	24,032	60	16,510
56	22,393	61	15,203
57	20,822	62	13,960
58	19,318	63	12,781
59	17,881	64	11,663

Interest rate: 3.0% per year, compounded annually.

X = the annual early retirement benefit commencing at age 58 payable under the optional form of benefit.

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Question 28

In what range is X ?

- (A) Less than \$6,800
- (B) \$6,800 but less than \$7,000
- (C) \$7,000 but less than \$7,200
- (D) \$7,200 but less than \$7,400
- (E) \$7,400 or more

Data for Question 29 (2 points)

Smith retires at age 60 and is offered the following distribution options:

Option 1 \$90,000 lump sum, payable at age 60

Option 2 \$100,000 lump sum, payable at age 62, if Smith is alive at age 62

Mortality: $\ell_x = 100 - x$, $0 \leq x \leq 100$

Interest rate: 5.0% per year, compounded annually.

X = the absolute value of the difference in the present values of the two options.

Question 29

In what range is X ?

- (A) \$0
- (B) \$1 but less than \$1,500
- (C) \$1,500 but less than \$3,000
- (D) \$3,000 but less than \$4,500
- (E) \$4,500 or more

Data for Question 30 (4 points)

Smith (age 40) takes out a special life insurance policy that provides the following:

Death benefit: \$100,000

If Smith dies prior to age 65, the death benefit is payable when Smith would have attained age 65.

If Smith dies subsequent to age 65, the death benefit is payable at the end of the year of death.

Interest rate: 3.0% per year, compounded annually

Selected commutation functions:

\underline{x}	\underline{D}_x	\underline{N}_x
40	2,522	45,846
...
65	722	6,782

X = the single premium at age 40 for this policy.

Question 30

In what range is X ?

- (A) Less than \$42,000
- (B) \$42,000 but less than \$44,000
- (C) \$44,000 but less than \$46,000
- (D) \$46,000 but less than \$48,000
- (E) \$48,000 or more

Data for Question 31 (3 points)

Selected mortality rates:

$$q_{65} = 0.10$$

$$q_{66} = 0.12$$

Deaths are uniformly distributed between consecutive integral ages.

$$X = {}_{0.75}q_{65.50}$$

Question 31

In what range is X ?

- (A) Less than 0.073
- (B) 0.073 but less than 0.076
- (C) 0.076 but less than 0.079
- (D) 0.079 but less than 0.082
- (E) 0.082 or more

****END OF EXAMINATION****